Surgery methods in spectral geometry

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Abstract: A surgery is a topological modification of a manifold: one removes the neighborhood of an embedded sphere with trivial normal bundle, i.e. a set of the form $S^k \times B^{n-k}$ and then one glues in $B^{k+1} \times S^{n-k-1}$. The simplest example is the operation of taking a connected sum ($k = 0$). We study the behavior of the spectrum of the Dirac operator and of the conformal Laplacian under surgeries. The manifold after surgery carries a metric that has similar spectral properties as the manifold before surgeries. A central method is to study eigenfunctions on generalized cylinders. In order to avoid technical difficulties we start the minicourse by studying simple examples, i.e. manifolds with long cylindrical parts. We will see that eigenfunctions of the Dirac operator and of the conformal Laplacian have a convex behavior on such cylinders. Such cylinders are used for taking connected sums. If we want to deal with arbitrary surgeries, then the notion of cylinder has to be generalized. Similar convex behavior also holds on such generalized cylinders.

Once, surgeries are understood, applications to bordisms follow, as a bordism can be split into a sequence of surgeries.

We will derive applications to index theory and to a problem in conformal geometry. The first conclusion will be that for generic metrics the kernel of the classical Dirac operator is as small as allowed by the index theorem, which was an open conjecture since Atiyah and Singer proved their index theorems in the 1960s. We will also see that the smooth Yamabe invariant is a bordism invariant.