Surgery methods in spectral geometry

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Main Collaborators
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Literature

Yamabe problem

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Dirac operators

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Bordism and Surgery

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Preliminary results

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Subject of the talks

- B. Ammann, P. Jammes, *The supremum of first eigenvalues of conformally covariant operators in a conformal class*

- B. Ammann, M. Dahl and E. Humbert, *Smooth Yamabe invariant and surgery*

- B. Ammann, M. Dahl and E. Humbert, *Surgery and harmonic spinors*

- B. Ammann, M. Dahl and E. Humbert, *Harmonic spinors and local deformations of the metric*

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Theorem 1. *Let Γ be a finitely presented group, $n \geq 5$. Let $W^{n+1} \to B\Gamma$ be a bordism between $M \to B\Gamma$, $M \neq \emptyset$ and $N \to B\Gamma$, $N \neq \emptyset$, such that*

$$\begin{array}{ccccc} M & \hookrightarrow & W & \hookleftarrow & N \\ & \searrow & \downarrow & \swarrow & \\ & & B\Gamma & & \end{array} \qquad (*)$$

commutes. Assume that $\pi_1(N) \to \pi_1(B\Gamma) = \Gamma$ is an isomorphism.
Also assume that either

(a) M, W, and N are spin, or that

(b) M, W, and N are oriented, and $\widetilde{N}$ does not admit a spin structure.

Then there is a bordism $\overline{W} \to B\Gamma$ between $M \to B\Gamma$ and $N \to B\Gamma$ with $(*)$ that decomposes into surgeries of dimensions $0, 1, \ldots, n-3$.